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Seismic-Response Peak Factors for Typical Highway Bridges Subjected to Uniform and Multiple-Support Excitations

A.S. Nazmy[1]

ABSTRACT

Response of a typical four-span continuous R.C. highway bridge to either uniform or spatially-varying seismic excitation is evaluated in the time and frequency domains. A time-history analysis has been employed first to compute the peak response values, then a stochastic approach based on the linear random vibration theory was utilized to determine the root mean square (R.M.S.) response values. The response peak factors were then computed for critical response quantities by comparing the R.M.S. values with the corresponding peak values. The stochastic analysis included the effects of both modal and support-excitation cross-correlation on the seismic response. The study provides a practical range of peak factors that can be used along with the stochastic analysis results for computing the peak values of seismic responses for typical continuous-girder R.C. highway bridges. It also indicates that the spatial variability of ground motion could have a significant effect on the dynamic response of highway bridges.

INTRODUCTION

It is important to design highway bridges to withstand major seismic events without major damage or complete collapse. This requires a good estimate of the level of bridge response (joint displacements and member forces) during any future earthquake. Analysis in the time domain, using time-history records from a past earthquake, determines the bridge response to that specific earthquake. However, the results could be misleading for design purposes since future earthquakes could have very much different time-history records. Analysis in the frequency domain, which is based on the random vibration theory, covers a wide spectrum of seismic records since it is based on the earthquake stochastic characteristics which could be the same for different earthquakes. Moreover, it can easily handle the effects of modal interaction when closely-spaced modes of vibration are encountered, and can provide a clear picture of the contribution of each mode of vibration to the total response, in addition to its computational advantage in dealing with product terms rather than the convolution integrals required in a time-domain analysis. However, the drawback in this analysis type is its prediction of the root mean square (R.M.S.) values rather than the peak responses which are needed for design. Therefore, if a practical range of peak factors (defined as the ratio between the peak and the R.M.S. responses) is known, these factors can be used with the stochastic analysis results to compute the peak responses needed for bridge seismic design.

Several researchers investigated in the past the response of multisupport structural systems to earthquakes using the random vibration approach (Abdel-Ghaffar and Rubin 1982 and 1984; Hao 1991; Tan 1994; Heredia-Zavoni and Vanmarcke 1994). Different mathematical models have been proposed to

[1]Associate Professor, Department of Civil and Environmental Engineering, University of Maine, 5711 Boardman Hall, Orono, ME 04469-5711, USA.

provide stochastic representation of the ground motion at the supports of multisupport structures taking into account the effects of coherency decay and seismic wave propagation on the degree of correlation between the support motions (Harichandran and Wang 1990; Perotti 1990; Zerva 1990). In addition, some attempts have been made recently to develop a modified response spectrum method for such multiply supported structures (Der Kiureghian and Neuenhofer 1992; Yamamura and Tanaka 1990). In most of these investigations, the support excitation was either in the vertical or the longitudinal direction of the structure without the simultaneous excitation effect being considered. In addition, no attempt has been made to provide a practical range of response peak factors to be used for the design purpose.

In the present study, a four-span continuous R.C. highway bridge was subjected first to uniform seismic excitation, and again to multiple-support excitation using the 1979 El Centro earthquake records. The effects of both modal and support-excitation cross-correlation were included in the analysis. Support motion was provided in the vertical and longitudinal directions of the bridge simultaneously, and the analysis was performed in both the time and frequency domains. Peak factors for both vibrational and total (vibrational plus pseudo-static) responses were then computed for different response quantities by comparing the R.M.S. values obtained from the frequency-domain analysis with the corresponding peak response values obtained from the time-domain analysis. It was the main objective of this study to provide a practical range of response peak factors to be used in the seismic design of typical continuous-span reinforced concrete highway bridges when subjected to either uniform or multiple-support seismic excitations under the simultaneous effect of vertical and longitudinal ground motions.

BASIS FOR COMPUTING RESPONSE DISPLACEMENTS AND FORCES

Equations of Motion for Multiple-Support Analysis

The equations of motion of any structural system subjected to uniform or multiple-support seismic excitation can be expressed in terms of the partitioned mass [M], damping [C], and stiffness [K] matrices as follows (Nazmy and Abdel-Ghaffar 1987; Clough and Penzien 1993)

$$\begin{bmatrix} M_{ss} & M_{sg} \\ M_{gs} & M_{gg} \end{bmatrix} \begin{Bmatrix} \ddot{u}_s \\ \ddot{u}_g \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sg} \\ C_{gs} & C_{gg} \end{bmatrix} \begin{Bmatrix} \dot{u}_s \\ \dot{u}_g \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sg} \\ K_{gs} & K_{gg} \end{bmatrix} \begin{Bmatrix} u_s \\ u_g \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{1}$$

where {u} denotes the total dynamic displacement vector; the subscript "g" denotes the degrees of freedom corresponding to the points of application and directions of ground motion; and the subscript "s" denotes all other degrees of freedom of the structure. A consistent mass matrix (Przemieniecki 1985) has been utilized in the present analysis to provide a high accuracy in the results.

Solution in the Time Domain

The total dynamic displacement of each joint may be decomposed into pseudo-static (subscript "p") and vibrational (subscript "v") displacements as follows:

$$\begin{Bmatrix} u_s \\ u_g \end{Bmatrix} = \begin{Bmatrix} u_{ps} \\ u_{pg} \end{Bmatrix} + \begin{Bmatrix} u_{vs} \\ 0 \end{Bmatrix} \tag{2}$$

For linear analysis, the above equation may be written as:

$$\begin{Bmatrix} u_s \\ u_g \end{Bmatrix} = \sum_{j=1}^{G} \begin{Bmatrix} g_{psj} \\ g_{pgj} \end{Bmatrix} f_j(t) + \sum_{n=1}^{N} \begin{Bmatrix} \phi_n \\ 0 \end{Bmatrix} q_n(t) \tag{3}$$

where g_{psj} represents the j^{th} pseudo-static function that results from unit displacement in the j^{th} degree of freedom at a supporting point; $f_j(t)$ is the ground displacement time history in the j^{th} degree of freedom at a support; g_{pgj} is a G x 1 vector with the j^{th} element equal to one and all its other elements being zero; "G" is the total number of ground motion inputs at the supports; $\{\phi_n\}$ is the n^{th} vibrational mode shape; $q_n(t)$ is the n^{th} generalized coordinate; and "N" is the number of mode shapes used in the modal analysis.

Upon substitution of Eq. (3) into Eq. (1), pre-multiplying the resulting equation by the transpose of the n^{th} vibrational mode shape and using the orthogonality of modes, and knowing that for an unloaded structure with a static condition of its supports $K_{ss}u_{ps}+K_{sg}u_{pg}=0$, we get the following governing equation for the n^{th} generalized coordinate (Nazmy and Abdel-Ghaffar 1987):

$$\ddot{q}_n(t)+2\zeta_n\,\omega_n\,\dot{q}_n(t)+\omega_n^2\,q_n(t)=\sum_{j=1}^{G}\left(\alpha_{nj}\,\ddot{f}_j(t)+\beta_{nj}\,\dot{f}_j(t)\right), \qquad n=1,2,3,\ldots.N \tag{4}$$

in which ζ_n and ω_n are the damping ratio and natural frequency, respectively, of the n^{th} vibration mode, while α_{nj} and β_{nj} are the modal participation coefficients. However, the contribution from the β terms in the above equation is often small and can be neglected. Thus Eq. (4) may be re-written as:

$$\ddot{q}_n(t)+2\zeta_n\,\omega_n\,\dot{q}_n(t)+\omega_n^2\,q_n(t)=\sum_{j=1}^{G}\left(\alpha_{nj}\,\ddot{f}_j(t)\right), \qquad n=1,2,3,\ldots.N \tag{5}$$

where,

$$\alpha_{nj}=\frac{-\{\phi_n\}^T\begin{bmatrix}M_{ss} & M_{sg}\end{bmatrix}\begin{Bmatrix}g_{psj} & g_{pgj}\end{Bmatrix}^T}{\{\phi_n\}^T[M_{ss}]\{\phi_n\}}, \quad j=1,2,\ldots.G, \qquad n=1,2,3,\ldots.N \tag{6}$$

Eq. (5) can be solved for $q_n(t)$ using the convolution integral, and Eq. (3) may then be used to determine both vibrational and total response displacements. Bending moments and shear forces in any member can be computed from its nodal displacement time histories and its stiffness properties. From displacement and member force time histories, the absolute maximum values can be determined.

Solution in the Frequency Domain

Using the random vibration approach, the power spectral density (PSD) function of the total response displacement $\{G_u\}$ can be obtained by multiplying the finite Fourier transform of the R.H.S. of Eq. (3) by its complex conjugate and then by $(2/T_1)$, where T_1 is the time duration of the ground motion input. This leads, after some math manipulation, to (Rubin et al. 1983):

$$\begin{Bmatrix} G_{us} \\ G_{ug} \end{Bmatrix} = \sum_{j=1}^{G}\sum_{k=1}^{G} \begin{Bmatrix} g_{psk} \\ g_{pgk} \end{Bmatrix}\begin{Bmatrix} g_{psj} & g_{pgj} \end{Bmatrix} G_{jk}(\omega) + \sum_{j=1}^{G}\left(\begin{Bmatrix} G_{uvsj} \\ 0 \end{Bmatrix} + \begin{Bmatrix} G_{juvs} \\ 0 \end{Bmatrix}\right)\begin{Bmatrix} g_{psj} & g_{pgj} \end{Bmatrix} + \begin{Bmatrix} G_{uvs} \\ 0 \end{Bmatrix} \tag{7}$$

where,

$$G_{jk}(\omega) \cong \frac{2}{T_1}\overset{*}{F_j}(\omega)\,F_k(\omega) \tag{8}$$

$$\{G_{uvsj}\} \cong \frac{2}{T_1}\{\overset{*}{U}_{vs}(\omega)\}F_j(\omega) = \frac{2}{T_1}\sum_{n=1}^{N}\{\phi_n\}\{\overset{*}{H}_n(\omega)\}^T\{\overset{*}{F}(\omega)\}F_j(\omega)\ ,\quad j=1,2,\ldots G \tag{9}$$

$$\{G_{juvs}\} \cong \frac{2}{T_1}\overset{*}{F}_j(\omega)\{U_{vs}(\omega)\} = \frac{2}{T_1}\sum_{n=1}^{N}\{\phi_n\}\overset{*}{F}_j(\omega)\{H_n(\omega)\}^T\{F(\omega)\}\ ,\quad j=1,2,\ldots G \tag{10}$$

$$\{G_{uvs}\} \cong \frac{2}{T_1}\{\overset{*}{U}_{vs}(\omega)\}\{U_{vs}(\omega)\}^T = \sum_{n=1}^{N}\sum_{m=1}^{N}\{\phi_n\}\{\phi_m\}^T\{\overset{*}{H}_n(\omega)\}^T[G_{ff}(\omega)]\{H_m(\omega)\} \tag{11}$$

In the above equations, $U_{vs}(\omega)$ and $F_j(\omega)$ are Fourier transforms of the vibrational response displacement $u_{vs}(t)$ and the ground displacement $f_j(t)$, respectively; $\{H_n(\omega)\}$ is vector of the n^{th} complex frequency response functions (or transfer functions); and the superposed asterisk denotes the complex conjugate.

$\{G_{uvs}\}$ given by Eq. (11) is the PSD function of the vibrational response displacement, while $G_{jk}(\omega)$ given by Eq. (8) is the cross spectral density function between the ground inputs $f_j(t)$ and $f_k(t)$. The diagonal elements of the spectral matrix $[G_{ff}(\omega)]$ in Eq. (11), when j=k, correspond to the PSD of the j^{th} displacement input $f_j(t)$, while the off-diagonal elements correspond to the cross-spectral densities. In earthquake-response analysis of multisupport structural systems, such as bridges, the various input motions at the supports originate from the same source, and are therefore correlated. In such case, the off-diagonal terms in the spectral matrix of excitation $[G_{ff}(\omega)]$ are non-zeros. The above formulation thus takes into account the cross-correlation between input motions. It also takes into account the modal interaction between the natural mode shapes of free vibration of the bridge, which is accounted for by the double summation in Eq. (11).

By integrating the PSD functions of the vibrational and total response displacements over the entire frequency range, the corresponding mean square values, and consequently the R.M.S. values, can be determined. With regard to member end forces, Fourier transforms of the vibrational and total member forces for any element in the bridge can be computed using Fourier transforms of its vibrational and total joint displacements, respectively, along with its linear stiffness properties. Hence, the PSD functions, the mean square values and the R.M.S. values of member end forces can be computed.

THE ANALYTICAL MODEL

Figure (1) shows an elevation of the analytical model used in this study along with the nodal numbering system used for the finite element discretization. The model, which has been adapted after Léger et al. (1990), represents a typical four-span continuous reinforced concrete (R.C.) highway bridge supported on two roller supports at the abutments and hinged supports at the column bases. This supporting system is typical for highway bridges constructed in seismically active zones. The deck elements are prismatic with cross-sectional area of 43650 cm^2, moment of inertia of 17.99x10^7 cm^4, and distributed mass of 11350 kg/m. The properties for the bridge piers (or columns) are 20110 cm^2 for the cross-sectional area, 32.17x10^6 cm^4 for the moment of inertia, and 5228 kg/m for the distributed mass. A Young's modulus of 25000 MPa was used for all the elements, and a damping ratio of 5% of the critical value was used for all the modes of vibration.

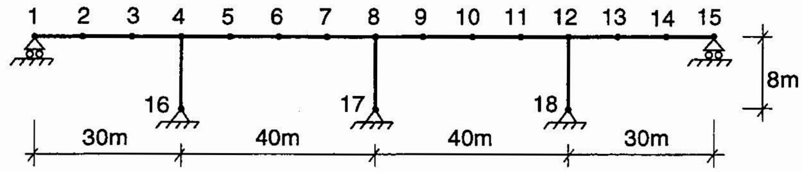

Figure 1. The analytical model.

GROUND MOTION INPUTS

Multiple-support seismic input can be represented by existing strong motion records. In the present study, selective ground motion records from the Imperial Valley, CA, (El Centro) earthquake of October 15, 1979 were employed in defining the uniform and multiple-support input motions since the ground accelerations of these records are rich in high frequency components and the recording stations were closely spaced. Seismic records from 5 different stations were applied at the 5 supports of the bridge for the multiple-support excitation case. For the uniform input case, the record that contained the highest peak ground acceleration was applied at all 5 supports. Vertical components only were applied at the abutments, while vertical and longitudinal components were applied simultaneously at the column bases. The time histories of these records were used for the time-domain analysis, while their finite Fourier transforms were used in the frequency-domain analysis.

EARTHQUAKE-RESPONSE AND PEAK FACTORS

Two computer programs have been developed for performing the time and frequency domain analyses described earlier. The time domain analysis program gives the time histories and absolute maximum values of vibrational and total response displacements and member forces for selected joints and members in the model. The frequency domain analysis program provides the R.M.S. values of selected response displacements and member forces. Both programs can provide the bridge response to either uniform or multiple-support seismic excitations. In addition, they can also provide among the output the free vibration characteristics of the bridge model.

Fig. (2) shows the response time histories of the vertical displacement of joint 10 (at the middle of the interior span), and the longitudinal displacement of joint 15 (at the roller support) for the cases of uniform and multiple-support excitations. Only the vibrational (or relative) displacements are shown in this figure since the total (vibrational plus pseudo-static) displacement responses are not important for design purpose. It can be noticed in this figure that the vertical displacement of joint 10 has decreased by almost 35% for the multiple-input case in comparison with the uniform input case. This is due to the out-of-phase movement at the supports in the former case, which excited the anti-symmetric bending modes with minimal contribution to the vertical movement of joint 10. However, the figure also shows that the maximum longitudinal displacement at the roller support (joint 15) is almost the same for the cases of uniform and multiple-support excitation.

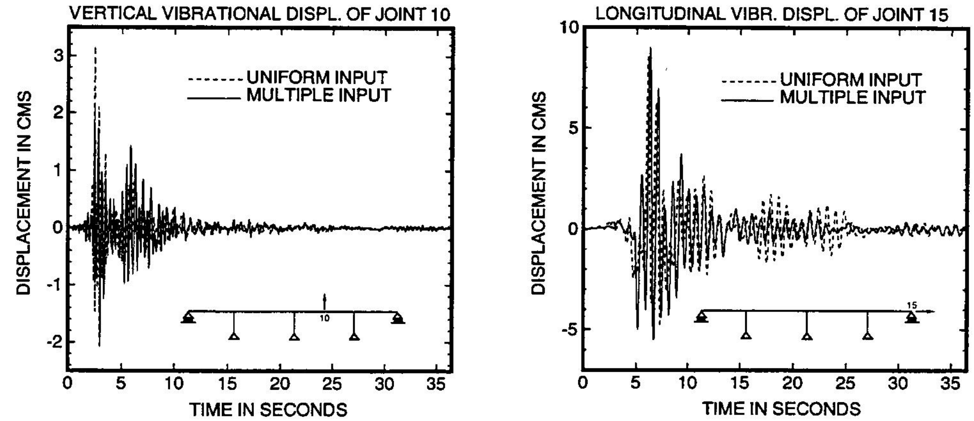

Figure 2. Time histories of the vibrational response displacements at joints 10 and 15 for the cases of uniform and multiple-support excitations.

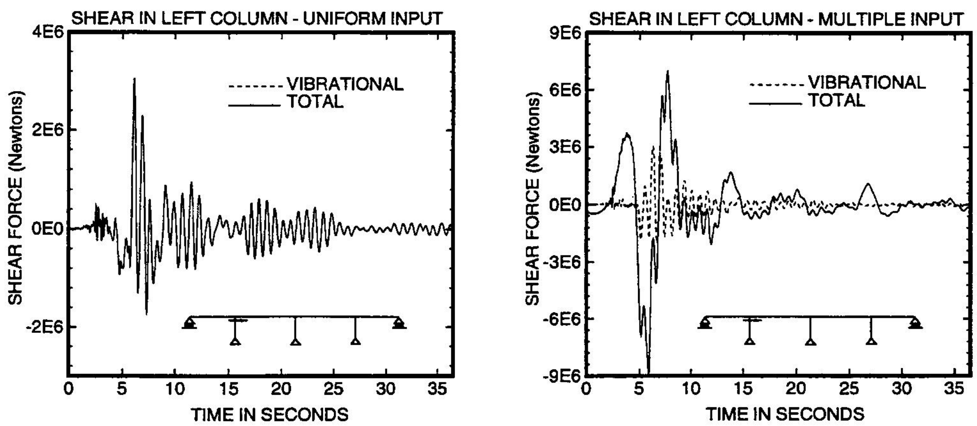

Figure 3. Time histories of the response shear force in the left column for the cases of uniform and multiple-support excitations.

Bending moments and shear forces in the left and middle columns of the bridge were selected for observation is this study, since bridge designers are mostly concerned with these parameters in earthquake-resistant design of highway bridges. Fig. (3) shows a comparison between the shear force in the left column for the cases of uniform and multiple-support excitations. It can be seen in this figure that the vibrational and total response curves are identical for the uniform input case, where the pseudo-static displacement is in fact a rigid body motion. However, for the multiple-support input case the pseudo-static displacement caused relative movement between the column ends and produced a large increase in the shear force (the absolute maximum value almost tripled).

To compute the seismic response peak factors, each of the computed absolute maximum responses (vibrational and/or total) obtained from the time-history analysis was divided by the corresponding R.M.S. response obtained from the frequency domain analysis. Table (1) lists the absolute maximum value, the R.M.S. value, and the corresponding peak factor for selected response displacements and member forces for both cases of uniform and multiple-support excitations. It can be noticed in this table that the peak factors for the vibrational displacement responses varied between 6.4 and 6.5 in the uniform input case, and between 6.6 and 6.8 in the multiple-input case. For shear forces and bending moments in the bridge piers, the peak factors in the uniform input case varied between 6.5 and 6.6, while in the multiple-input case they varied between 6.3 and 7.1 (for the vibrational responses) and between 4.6 and 6.5 (for the total response). Therefore, for design purpose a response peak factor of 6-7 may be used for vibrational displacements, and for moment or shear in the bridge piers in the uniform input case. In the multiple-input case, a peak factor of 4.5-6.5 may be used for the pier total forces.

Table 1: Response peak factors for uniform and multiple-support excitations.

Response Quantity	Response to Uniform Input			Response to Multiple Input		
	Absol. max. value	R.M.S. value	Peak factor	Absol. max. value	R.M.S. value	Peak factor
Vertical displ. of joint 10 (cm)	3.14 (vibr.)	0.49 (vibr.)	6.4	2.04 (vibr.)	0.30 (vibr.)	6.8
Vertical displ. of joint 13 (cm)	2.17 (vibr.)	0.34 (vibr.)	6.4	1.72 (vibr.)	0.26 (vibr.)	6.6
Long. displ. of joint 15 (cm)	8.55 (vibr.)	1.31 (vibr.)	6.5	8.99 (vibr.)	1.33 (vibr.)	6.8
Shear force in the left column (Newton)	305.5×10^4 (vibr.) 305.5×10^4 (total)	47.2×10^4 (vibr.) 47.2×10^4 (total)	6.5 6.5	303.9×10^4 (vibr.) 896.1×10^4 (total)	48.2×10^4 (vibr.) 193.9×10^4 (total)	6.3 4.6
Bending Moment at the top of the left column (N-m)	245.2×10^5 (vibr.) 245.2×10^5 (total)	37.9×10^5 (vibr.) 37.9×10^5 (total)	6.5 6.5	244.1×10^5 (vibr.) 717.7×10^5 (total)	38.7×10^5 (vibr.) 155.2×10^5 (total)	6.3 4.6
Shear force in the middle column (Newton)	336.6×10^4 (vibr.) 336.6×10^4 (total)	51.0×10^4 (vibr.) 51.0×10^4 (total)	6.6 6.6	374.4×10^4 (vibr.) 595.6×10^4 (total)	52.9×10^4 (vibr.) 91.1×10^4 (total)	7.1 6.5
Bending Moment at top of the middle column (N-m)	270.2×10^5 (vibr.) 270.2×10^5 (total)	40.9×10^5 (vibr.) 40.9×10^5 (total)	6.6 6.6	300.4×10^5 (vibr.) 477.4×10^5 (total)	42.5×10^5 (vibr.) 72.9×10^5 (total)	7.1 6.5

It should be mentioned at this point that the above recommended response peak factors are valid only for typical highway bridges, with span lengths in the range of 20 to 60 meters. However, for special long-span bridges such as suspension, cable-stayed, or arch bridges different values should be specified. Investigations for determining these values are beyond the scope of the present study.

CONCLUSIONS AND RECOMMENDATIONS

1. In order to obtain realistic earthquake-response quantities for design purposes, it is necessary to perform the analysis with the two orthogonal components of earthquake motion acting simultaneously at all supporting points of the bridge.

2. For typical highway bridges, the effect of multiple-support seismic excitation should be considered in the analysis, since the pseudo-static stresses or member forces are very significant.

3. In the earthquake-response analysis of bridges, the random vibration approach is a more powerful analytical tool than the time history analysis, since it provides a statistical measure of the response which is not controlled by an arbitrary choice of the input motion. It is particularly appealing from the viewpoint of design, where consideration must be given to entire families of potential ground motions.

4. It is very important, for design purposes, to establish a practical range of response peak factors for typical highway bridges to be used along with the results of the frequency domain analysis since this analysis provides only the R.M.S. response values. A factor between 6 and 7 may be used for the vibrational displacement responses in either the uniform or multiple-support excitation cases. For the total moment or shear in the bridge piers, a factor between 6 and 7 may still be used in the uniform input case, while a factor between 4.5 and 6.5 may be used in the multiple-input case.

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